range of variation of the wave numbers as large as desired.

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INVARIANT SOLUTIONS OF THE EQUATIONS OF THE NON-ISOTHERMAL STATIONARY FLOW OF A VISCOUS FLUID IN TUBES*

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The group properties /1/ of a system of equations describing flows in tubes of fluids the viscosity of which depends on the temperature are investigated for large Peclet numbers. It is shown that for exponential and power dependences there is an extension of the main group of transformations. For these cases, invariant solutions which have a physical meaning are considered.

The equations describing the motion of a viscous fluid in a cylindrical tube may be written, in dimensionless form as follows for $\delta \ll 1, \mathrm{Pe} \gg 1$ /2/:

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$$\frac{\partial p}{\partial R}=0,\quad \frac{\partial p}{\partial z}=\delta\,\mathrm{Pe}\,\frac{1}{R}\,\frac{\partial}{\partial R}\,(\mu R u) \tag{1}$$

$$\frac{\partial v}{\partial R}+R\,\frac{\partial u}{\partial z}=0,\quad \frac{\partial^2 T}{\partial R^2}+\frac{1-v}{R}\,\frac{\partial T}{\partial R}=u\,\frac{\partial T}{\partial z} \tag{2}$$

Here

$$z=\frac{x}{\mathrm{Pe}\,r_0},\quad \mu=\frac{\eta}{\eta_0},\quad T=\frac{t}{t_0},\quad R=\frac{r}{r_0},\quad \delta=\frac{r_0}{l},\quad p=\frac{P}{P_0}$$

$$v=R\,\frac{V_r\,\mathrm{Pe}}{2V_0},\quad u=\frac{V_x}{2V_0},\quad P_0=\frac{2\eta_0 l V_0}{r_0^2},\quad \mathrm{Pe}=\frac{2V_0 r_0}{a}$$

where x is a longitudinal coordinate, r is the distance from the tube axis, r_0 is the radius of the tube, t is the temperature, V_x and V_r are (respectively) the longitudinal and the radial components of the velocity, η is the viscosity of the fluid, l is the length of the tube, P is the pressure, t_0, η_0 and V_0 are the characteristic values of the temperature, viscosity and velocity and Pe is the Peclet number.

It follows from the first equation of (1) that $\partial p/\partial z$ is some function of z, which we will denote by $g(z)$. Thus, the second equation of (1) may be integrated once with respect to R with the natural symmetry condition $\partial u/\partial R/_{R=0}=0$. Introducing the notation $f(T)=\delta\mu\mathrm{Pe}/2$, Eq.(1) may be replaced by the following

$$\partial u/\partial R = Rf(T)\,g(z) \tag{3}$$

We carry out the group classification /1/ of system (2), (3).

For an arbitrary form of the function f, the system admits of an infinitesimal operator:

$$X_1=R\left(1-z\,\frac{g'}{g}\right)\frac{\partial}{\partial R}+4z\,\frac{\partial}{\partial z}+2u\left(1+z\,\frac{g'}{g}\right)\frac{\partial}{\partial u}-R^2u\left(z\,\frac{g'}{g}\right)\frac{\partial}{\partial v}$$

$$X_2=-R\,\frac{g'}{g}\,\frac{\partial}{\partial R}+4\,\frac{\partial}{\partial z}+2u\,\frac{g'}{g}\,\frac{\partial}{\partial u}-R^2u\left(\frac{g'}{\partial}\right)'\frac{\partial}{\partial v}$$

An extension of this algebra is obtained for the following specifications of $f(T)$ apart from an equivalence transformations /1/:

1) $f(T)\equiv\mathrm{const}$; additional basis operators

$$X_3=\partial/\partial T,\quad X_4=T\partial/\partial T$$

2) $f(T)=T^{\gamma}$; additional operator

$$X_5=\gamma R\partial/\partial R-4T\partial/\partial T+2\gamma u\partial/\partial u$$

3) $f(T)=e^{T}$; additional operator

$$X_6=R\partial/\partial R-4\partial/\partial T+2u\partial/\partial u$$

We consider a number of invariant solutions corresponding to these operators, which have a physical interpretation.

For

$$f(T)=e^{\varepsilon T},\quad g(z)=-2p_0e^{-\varepsilon z},\quad p_0=\mathrm{const}$$

an invariant solution of the operator X_2-X_6 has the form

$$v=\varphi_1(R),\quad u=\varphi_2(R),\quad T=z+\varphi_3(R)$$

φ_i $(i=1,2,3)$ satisfy a system of ordinary differential equations, the solution of which for the boundary conditions

$$v/_{R=1}=u/_{R=1}=0$$

may be written in the form

$$\varphi_1=0,\quad \varphi_k=\varphi_k^{(0)}+\varepsilon\varphi_k^{(1)}+O(\varepsilon^2),\quad k=2,3$$

$$\varphi_2^{(0)}=p_0(1-R^2),\quad \varphi_2^{(1)}=-2p_0\int_R^1 R\varphi_3^{(0)}dR$$

$$\varphi_3^{(i)}=\int_0^R\left(\int_0^R R\varphi_2^{(i)}dR\right)\frac{dR}{R}+\alpha,\quad \alpha\equiv\mathrm{const},\quad i=0,1$$

For

$$f(T)=e^{\varepsilon T},\quad g(z)=-2p_0z^{1-\varepsilon},\quad p_0=\mathrm{const}$$

an invariant solution of the operator X_1-X_6 has the form

$$v=\varphi_1(R),\quad u=z\varphi_2(R),\quad T=\ln z+\varphi_3(R)$$

where the functions φ_i $(i=1,2,3)$ satisfy the system of ordinary differential equations

$$\varphi_2' = -2p_0 R e^{\varepsilon\varphi_3}, \quad R\varphi_2 + \varphi_1' = 0$$
$$(1-\varphi_1)\varphi_3' + R\varphi_3'' - R\varphi_2 = 0$$

the solution of which

$$\varphi_i = \varphi_i^{(0)} + \varepsilon\varphi_i^{(1)} + O(\varepsilon^2), \quad i = 1, 2, 3$$
$$\varphi_1^{(0)} = P_0(R^4/4 - R^2/2), \quad \varphi_2^{(0)} = p_0(1-R^2), \quad \varphi_3^{(0)} =$$
$$p_0\left(\int_R^1 \left(\int_0^R (1-R^2) RF(R)\,dR\right)\frac{dR}{RF(R)} + \alpha\right), \quad \varphi_1^{(1)} = -\int_R^1 R\varphi_2^{(1)} dR$$
$$\varphi_2^{(1)} = -2p_0 \int_R^1 R\varphi_3^{(0)} dR, \quad \varphi_3^{(1)} = \int_R^1 \left(\int_0^R (R\varphi_2^{(1)} + \varphi_1^{(1)}\varphi_3^{(0)\prime}) RF(R)\,dR\right)\frac{dR}{RF(R)}$$
$$F(R) = \exp((p_0/4)(R^2 - R^4/4))$$

describes a flow in a tube with permeable walls for a constant rate of injection (suction) $v_{R=1} = -p_0/4$.

For arbitrary functions f and g an invariant solution of the operator X_2 may be written in the form

$$v = -\frac{1}{4}\frac{g'}{g}\varphi(\xi), \quad u = \frac{\varphi(\xi)}{R^2}, \quad T = A_1 \ln\xi + A_2$$
$$\varphi = \xi^{1/2}\left({}^1/_4 \int \xi^{-1/2} f(T)\,d\xi + A_3\right), \quad \xi = R^4 g(z)$$

where A_i $(i = 1, 2, 3)$ are arbitrary constants, which may be adjusted so that

$$\varphi(\beta_i) = 0, \quad \beta_i > 0, \quad i = 1, 2$$

This solution corresponds to a flow in an annular channel, the radius of the walls of which varies as $R_i = (\beta_i/g)^{1/4}$. Since $g(z)$ is an arbitrary function and the initial system of equations is invariant under shifts in z, we may choose a function $g(z)$ and a range of variation of z such that R_i is practically constant.

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DIFFRACTION OF SHEAR WAVES BY AN ELASTIC CYLINDRICAL INCLUSION WITH TWO CUTS ON THE PHASE BOUNDARY*

K.P. BELYAYEV

A method /1/ similar to that used in the case of one cut /2/ is used to determine the stress and deformation at the boundary of a cylindrical inclusion with two cuts placed on the contact contour. The external perturbation varies sinusoidally and is a plane wave in an isotropic medium. At the boundary of the inclusion the shear wave is reflected as a shear wave.

1. Formulation of the problem. Using a cylindrical system of coordinates we consider the effect of a plane shear wave on an elastic inclusion in the form of a circular cylinder $r \leqslant a, z \in (-\infty, \infty)$, bonded elastically along the edge $r = a$, $\theta \in \Omega = (\alpha_1, \pi - \alpha_2) \cup (\pi + \alpha_2, 2\pi - \alpha_1)$, $z \in (-\infty, \infty)$ where the area $r = a$, $\theta \in \Omega_0$, $(\Omega_0 = [-\alpha_1, \alpha_1] \cup [\pi - \alpha_2, \pi + \alpha_2])$ corresponds to two cuts

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